Predictive control to prevent wheel slippage and avoid obstacles on low friction roads in autonomous cars

Karthick Raja Dhavamani, Rishi Khajuriwala

***Abstract*—The development of autonomous cars depends on tackling many driving problems. Control of vehicles in low friction surfaces is one such problem. In this project, we are trying to solve this by implementing Model Predictive Control (MPC) by avoiding wheel slippage. Specifically, we will implement non-linear Gain-Scheduling MPC using MATLAB Model Predictive Control Toolbox. We will also be incorporating obstacle avoidance as constraints in our work.**

I. Introduction

In recent years, the interest in autonomous cars has increased exponentially. This has led to many new problems which were trivial for human drivers. Some of the problems that should be tackled in order to control an autonomous cars are lane following, obstacle avoidance, traffic regulations etc. Many of these problems are addressed already in prior works[1]]. One such problem of controlling vehicles on slippery roads is being studied in this work.

The prior works by Falcone et al. [2]-[5] have implemented MPC's for low friction roads. In 2010, Gao et al.[6] proposed a novel approach of obstacle avoidance on slippery roads by predictive control. In 2017, Basrah et al.[7] proposed yet another approach to be used in electric cars. In that paper, they have used linear and non-linear MPC's to control wheel slippage with torque blending between electric and hydraulic brakes.

In this paper, we will derive the dynamic model of the vehicle along with the contraints based on Pacejka's magic formula. We will apply non-linear Gain-Scheduling MPC strategy to control the vehicle on low friction roads with obstacle avoidance as one of the constraints. A feasibility analysis will be made to check the ranges of vehicle speeds and friction coefficients for which the controller can be used. A stretch goal for this project would be to linearize the above mentioned system and apply linear MPC to provide a comparison between them.

II. Preliminaries

In this paper, we will be deriving the dynamics of the system. Formulate constraints to avoid wheel slippage in low friction roads. Incorporate the constraints for obstacle avoidance within the system. Implement a non-linear controller design which can effectively solve the problem of wheel slippage and obstacle avoidance.

III. Formal Problem Statement

Control of vehicles in slippery roads is a major problem for autonomous cars. When combined with obstacle avoidance, this becomes a very important aspect for controlling the autonomous cars. Hence, we are trying to solve these problems by using Model Predictive Control Strategies.

IV. A Tentative Approach

In this project, we will be implementing a non-linear gain-scheduling model predictive control. The cost function would incorporate both wheel slippage and obstacle avoidance as constraints. Wheel slippage constraints will be formulated using Pacejka's magic formula. The cost functions would be optimised using MATLAB's Model Predictive Control Toolbox. The non-linear problem will be solved by using gain-scheduling MPC design.

V. Goals

Goals:

- Deriving the dynamics of the system
- Formulating the constraints for obstacle avoidance
- Choosing an appropriate cost function
- Implementing non-linear Gain-Scheduled MPC using MATLAB Model Predictive Control Toolbox
- Simulating the results in MATLAB
- Real-life feasibility analysis

Stretch Goals:

- Linearization of the system and constraints
- Implementation of linear MPC
- Comparison of Non-linear and Linear MPC
- Testing the Controller in RC Car available in CIBR lab

VI. Project Progress

Nomenclature

a	distance of front wheel from center of gravity
α	slip angle
B	Stiffness Factor
b	distance of rear wheel from center of gravity
β	side slip
C	Shape Factor
D	Peak Factor
δ	wheel steering angle
Δu	input signal changes of prediction model
E	Curvature
η	prediction model outputs
$(.)_f$	front wheel
$(.)_{ref}$	reference tracking signals
$f_c(.)$	functions describing lateral tire model
$f_l(.)$	functions describing longitudinal tire model
F_l, F_c	longitudinal and lateral tire forces
F_x, F_y	forces in car body frame
F_z	normal tire load

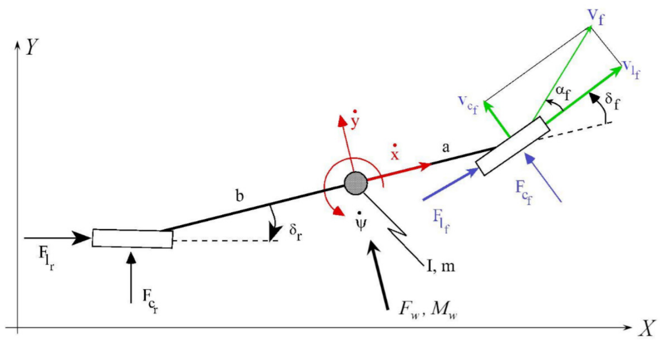

Fig. 1. Half car model

g	gravitational constant
H_p	output prediction horizon
H_u	control prediction horizon
I	car inertia
J	cost function
$(.)_k$	time step
k	slip ratio
$\underline{(.)}$	lower limit on variables
$\overline{(.)}$	upper limit on variables
m	car mass
μ	road friction coefficient
ψ	heading angle
Q, R	MPC weighting matrices
$(.)_r$	rear wheel
$\hat{(.)}$	predicted variables
r	wheel radius
s	slip ratio
T_s	sampling time
u	input signals of the prediction model
v_l, v_c	longitudinal and lateral wheel velocities
$\dot{x}$	vehicle speed
ξ	prediction model states
X, Y	absolute car position inertial coordinates
x, y	lateral,longitudinal coordinates in car body frame

- **Deriving the dynamic model:**
- *The half car model:*

  We will be using the half car model(bicycle model) to arrive at the lateral dynamics of the car. The equations that are used in this project are referred, analyzed and modified from previous research papers[8].The normal loads acting on the tires are considered to be constant. A simple model of the vehicle is depicted in Fig.1. The longitudinal, lateral and turning or yaw degrees of freedom(DOF) are given by

$$m\ddot{x} = m\dot{y}\dot{\psi} + F_{x_f} + F_{x_r} \tag{1}$$

$$m\ddot{y} = -m\dot{x}\dot{\psi} + F_{y_f} + F_{y_r} \tag{2}$$

$$I\ddot{\psi} = aF_{y_f} + bF_{y_r} \tag{3}$$

The vehicle's equations of motion in an absolute inertial frame are given by

$$\dot{Y} = \dot{x}\,sin(\psi) + \dot{y}\,cos(\psi) \tag{4}$$

$$\dot{X} = \dot{x}\,cos(\psi) - \dot{y}\,sin(\psi) \tag{5}$$

- *Tire model:*

  Longitudinal and lateral tire forces lead to the following forces acting on the center of gravity:

$$F_y = F_l\,sin(\delta) + F_c\,cos(\delta) \tag{6}$$

$$F_x = F_l\,cos(\delta) - F_c\,sin(\delta) \tag{7}$$

Tire forces for each tire are given by:

$$F_l = f_l(\alpha, s, \mu, F_z) \tag{8}$$

$$F_c = f_c(\alpha, s, \mu, F_z) \tag{9}$$

The slip ratio s is defined as

$$s = \begin{cases} \frac{rw}{v} - 1, & \text{if } v > rw, v \neq 0 \text{ for braking} \\ 1 - \frac{v}{rw}, & \text{if } v < rw, w \neq 0 \text{ for driving} \end{cases} \tag{10}$$

The slip angle is given by,

$$\alpha = \tan^{-1}\frac{v_c}{v_l} \tag{11}$$

Here wheel velocities are given by,

$$v_l = v_y\,sin(\delta) + v_x\,cos(\delta) \tag{12}$$

$$v_c = v_y\,cos(\delta) - v_x\,sin(\delta) \tag{13}$$

and

$$v_{y_f} = \dot{y} + a\dot{\psi}, \quad v_{y_f} = \dot{y} - b\dot{\psi} \tag{14}$$

$$v_{x_f} = \dot{x}, \qquad v_{x_r} = \dot{x} \tag{15}$$

The front and rear wheel normal forces are given by,

$$F_{z_f} = \frac{bmg}{2(a+b)} \tag{16}$$

$$F_{z_r} = \frac{amg}{2(a+b)} \tag{17}$$

- *The Simplified Pacejka's Tire model:*

  To compute the values of F_l and F_c given in equations (8) and (9) we can use the simplified empirical Pacejka's magic formula (18).

$$y(x) = D{*}sin\big[C{*}arctan\{Bx - E(Bx - arctan(Bx))\}\big] \tag{18}$$

and

$$Y(X) = y(x) + S_v \tag{19}$$

$$x = X + S_h \tag{20}$$

where

$$Y = F_l, F_c \; and \; X = \alpha \; or \; k \tag{21}$$

The values that we need to plug into these equations are mainly four constants which are determined empirically through experiments for different road surfaces. From

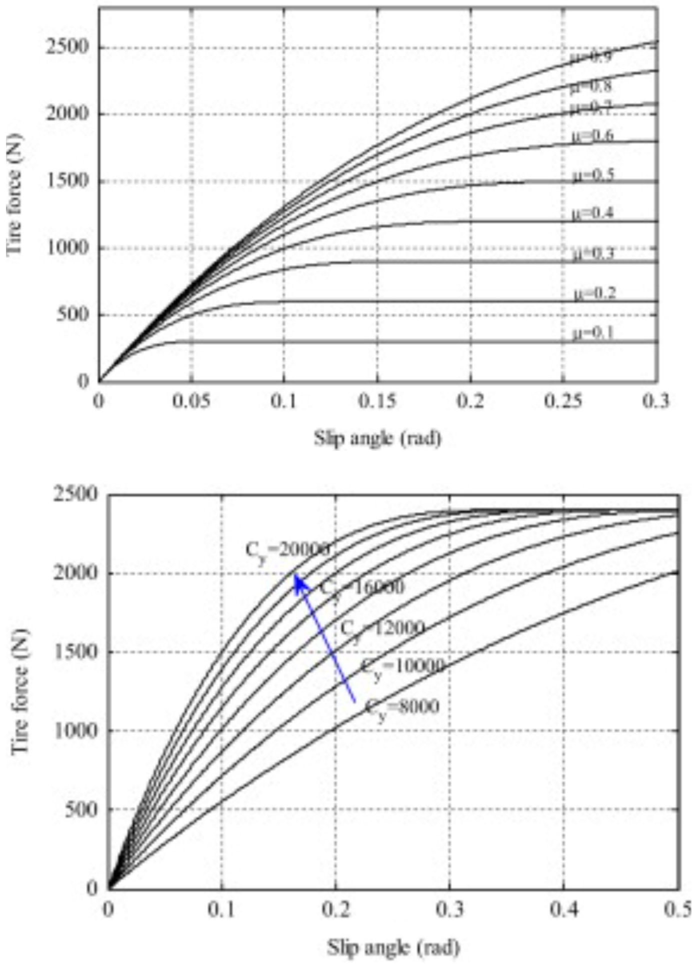

Fig. 2. Relationship between lateral force and slip angle. Source:[9]

the arrived values we can then calculate the cornering stiffness of the tires. We assume that the coefficient of friction is constant. Further simplifying, we can consider only the linear relationship region of F_c and slip angle, α obtained by Pacejka's formula to find the cornering stiffness by the following formula.

$$F_c = C_\alpha * \alpha \tag{22}$$

Thus, we can use substitute various values of cornering stiffness in the equations (1) to (3) to account for different road frictions.

- *Defining the states, inputs and output maps:*
  Using the above equations, we can define the state equation to be

$$\dot{\xi} = f_{s,\mu}(\xi, u) \tag{23}$$

$$\eta = h(\xi) \tag{24}$$

where the state is given by,

$$\xi = [y, \dot{y}, \psi, \dot{\psi}, Y, X] \tag{25}$$

and the input is given by,

$$u = \delta_f \tag{26}$$

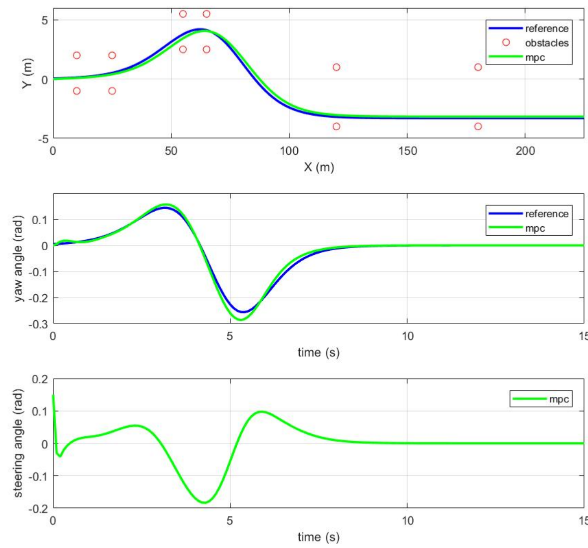

Fig. 3. Trajectory trajectory in MATLAB MPC Toolbox

Substituting the equations (1) - (17) and taking the stiffness of tire into account by Pacejka's magic formula, we can derive state matrix(A) and input matrix(B).
Thus, the output map can be defined as

$$h(\xi) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \xi \tag{27}$$

- **Formulating the constraints:**
  We are exploring different constraints that could be used for combining the obstacle avoidance, trajectory tracking and simultaneously avoiding slip. We are also trying to figure out the best cost function for the problem. Once we finalize the cost function we will start working on the feasibility analysis and report the time it takes to evaluate.

- **Simulation in MATLAB:**
  The cost function which incorporates only the trajectory tracking constraint is simulated in MATLAB with the help of Model Predictive Toolbox. The trajectory tracking is available in the MATLAB MPC toolbox as shown in the Fig. 3. We have modified the dynamics and added the state variables according to our derivation and tuned the parameters to track the trajectory.
  We will be changing the Obstacle positions in the Toolbox and check how it performs when obstacles are situated in along the reference trajectory for different cost functions.

VII. Questions

- Will we be able to solve non-linear MPC by Gain-scheduling method?

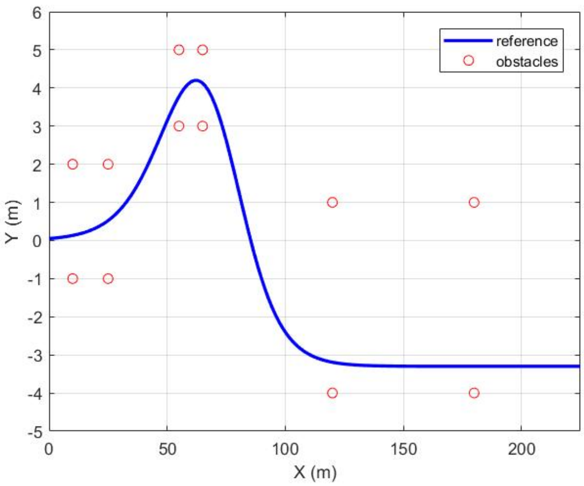

Fig. 4. Obstacle positions and reference trajectory

- Can the constraint of lane following be incorporated in this project?
- Will we be able to linearize the system analytically?
- Can we achieve a feasible MPC controller that could be used in real-life autonomous cars to avoid slippage?

REFERENCES

[1] T. S. A. Al-Zaher, A. M. Bayoumy, A. H. M. Sharaf and Y. H. H. El-din, "Lane tracking and obstacle avoidance for Autonomous Ground Vehicles," Mechatronics (MECATRONICS)

[2] Falcone, P., Borrelli, F., Asgari, J., Tseng, H. E., and Hrovat, D., 2007. Predictive active steering control for autonomous vehicle systems.

[3] Falcone, P., Tufo, F. B. M., Tseng, H. E., and Asgari, J.,2007. Predictive autonomous vehicles: A linear time varying model predictive control approach.

[4] Falcone, P., Borrelli, F., Asgari, J., Tseng, H. E., and Hrovat, D., 2007. A model predictive control approach for combined braking and steering in autonomous vehicles.

[5] Falcone, P., Borrelli, F., Tseng, H. E., Asgari, J., and Hrovat, D., 2007. Integrated braking and steering model predictive control approach in autonomous vehicles.

[6] Gao, Yiqi and Lin, Theresa and Borrelli, Francesco and Tseng, Eric and Hrovat, Davor. (2010). Predictive Control of Autonomous Ground Vehicles With Obstacle Avoidance on Slippery Roads.

[7] M. Sofian Basrah, Efstathios Siampis, Efstathios Velenis, Dongpu Cao and Stefano Longo (2017) Wheel slip control with torque blending using linear and nonlinear model predictive control, Vehicle System Dynamics

[8] Borrelli, F. , Falcone, P. , Keviczky, T. , Asgari, J. , Hrovat, D. (2005). MPC-based approach to active steering for autonomous vehicle systems. International Journal of Vehicle Autonomous Systems. 3, 24, 265-291 Abstract

[9] R. Wang, J. Wang, "Tire-road friction coefficient and tire cornering stiffness estimation based on longitudinal tire force difference generation", Control Engineering Practice, vol. 21, pp. 65-75, 2013.